On the Optimization of the Multiple Exponential Sweep Method*

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Measuring spatial features of sound sources and receivers is typically a time consuming task, especially when a high spatial resolution is required, as independent measurements have to be conducted for each measured direction. A speed-up in measurement time can be achieved with parallel measurement techniques using arrays of sound sensors or sources. For linear and time-invariant systems only loose restrictions are claimed for the excitation signal and the measurement method. Nevertheless, when measuring a sound receiver, e.g. directional microphones, the signals emitted by the multiple sound sources must be separable. Acoustic systems can be treated as linear systems for low input levels. However, when it comes to moderate levels, loudspeakers show non-linear behavior that cannot be neglected. To conduct a parallelized measurement technique at these levels the multiple exponential sweep method has recently been introduced to measure the acoustic transfer characteristics with weakly non-linear sound sources by using exponential sweeps. This method decreases the measurement time compared to sequential measurements. However, compared to the ideal linear case, the measurement duration is increased due to occurring harmonic impulse responses. A novel generalized overlapping strategy for these sweeps is proposed considering the length of each harmonic impulse responses and additionally the temporal structure of the desired impulse responses measured in anechoic environments. It is shown that the resulting optimized multiple exponential sweep method can yield even shorter measurement times than the original method.

0 INTRODUCTION

The measurement of transfer characteristics of acoustical systems is a common task and realized by using personal computers and digital signal processing. Systems are often assumed to be linear and time invariant (LTI) to apply the calculus introduced e.g. in system theory [1]. For this class of systems the correlation of the input and output signal is used to obtain the impulse response or—its Fourier transform—the transfer function. In the past decades, several different excitation signals have been studied regarding their performance in terms of signal to noise ratio (SNR), crest factor and measurement duration. They have been applied successfully always considering their different advantages and disadvantages. Maximum length sequences became popular due to a time and memory efficient algorithm leading directly to the impulse response of the system by using the fast Hadamard transform [2]. As the calculation time of impulse responses became less critical due to the improvement of the calculation complexity and speed of state-of-the-art personal computers sweep measurements gained popularity. Sweeps offer great advantages for systems that do not fully comply with the LTI assumption, e.g. weak time variances (slow changes of the system response over time) or non-linear transfer characteristics (harmonic distortion) [3]. In the last decade, a measurement method to identify harmonic distortion in loudspeakers based on exponential sweeps has been developed by MÜLLER ET. AL and FARINA [3, 4, 5]. Their research is mainly focussed on loudspeakers, known to be the most problematic element in the plackback and measurement chain, as they feature nonlinear behavior if driven with high levels due to their construction. The approach introduced by NOVAK uses these measurement results along with a non-linear system model to predict the behavior for different excitation levels [6].

The measurement of directional transfer functions or directivities of sensors or specific arrays of sensors has gained more interest during the last decade [7]. In the field of spatial or binaural recording and reproduction the need for a fast measurement procedure for directivities of individual persons rises. This can be explained by smaller localization errors of binaural signals generated by individual transfer functions [8]. The fast measurement of the transfer characteristics of multiple sound sources e.g. in a wave

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field synthesis application with more than 100 loudspeakers, can also be used to quickly monitor the status of each loudspeaker. A further application is the measurement of room impulse responses with certain directivity patterns using individually driven loudspeaker arrays. For a large number of sound sources the measurement duration rises and there is a need for a fast measurement procedure [9].

The parallel measurement e.g. of several loudspeakers, can be realized with pseudo-random sequences, since they are mutually orthogonal [10, 11]. These sequences show, however, disadvantages as the obtained impulse responses are very sensitive to time variances and non-linearities in the measurement chain [3].

Sweeps can also be used for multiple parallel excitation if the system can be approximated as an LTI system but certain deviation are allowed. MAJDAK ET. AL. introduced a novel fast measurement method for weakly nonlinear systems by using exponential sweeps and an optimization strategy to overcome interference in the measurement between non-linearities and the system's impulse responses [12]. Two different strategies to avoid this interference (overlapping and interleaving) were proposed and combined using an optimization algorithm yielding the so called *multiple exponential sweep method (MESM)*. MESM has been used to speed-up the measurement of HRTFs for 1550 directions and the whole procedure lasted for approx. 20 min [18].

In this paper we propose a generalized overlapping strategy which takes further advantage of a better understanding of the temporal structured of the impulse responses to be measured. The paper starts with a review of the sweep measurement technique and the original MESM, after which the proposed method and the mechanisms it is based on are described. It follows a comparison between the proposed method and the original MESM. It will be shown that under certain circumstances the generalized overlapping strategy of the sweeps yields even faster measurements times than the original MESM with unchanged accuracy. The paper concludes with two examples where the proposed method outperforms the original MESM.

1 REVIEW OF THE EXPONENTIAL SWEEP METHOD

The logarithmic magnitude of the impulse response of a weakly non-linear system measured with an exponential sweep is shown schematically in Fig. 1. Mostly one has interest in the fundamental impulse response located in the far right in this example. The impulse responses to the left of the fundamental impulse response are the harmonic impulse responses. Let τ_{IR} be the length of this fundamental impulse response, i.e. the time the system needs to decay into the noise floor. By using a monochromatic excitation non-linearities are observed as harmonics k. As for exponential sweeps it can be shown that the output signal consists of time and phase shifted exponential sweeps. These have the same sweep parameters as the excitation. The deconvolved result contains a fundamental impulse response and harmonics $h_{\mathrm{harm},k}$ with their length $\tau_{\mathrm{IR,k}}$. We define $\tau_{\mathrm{IR},1} = \tau_{\mathrm{IR}}$. Please note that, according to TORRAS-ROSELL AND JACOBSEN, nonlinearities also modify the fundamental impulse response and therefore the application of a time window to delete the harmonics does not entirely suppress the effects of nonlinearities in the measurement [13]. Details on sweep signal formulation, deconvolution and harmonics are presented in the appendix.

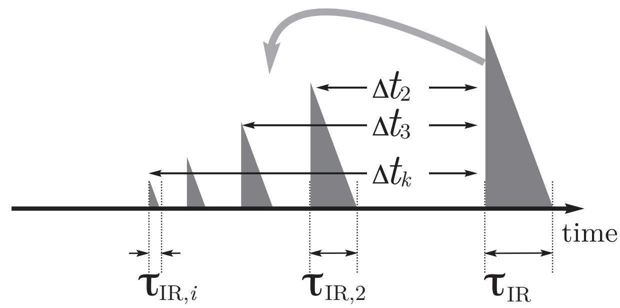

Fig. 1. Impulse response of a weakly nonlinear system obtained by exponential sweep measurement.

The time Δt_k between the fundamental impulse response and the harmonic k is given as

$$\Delta t_k = \frac{\log_2(k)}{r_{\mathrm{s}}}, \tag{1}$$

where r_{s} is the sweep rate for exponential sweeps that represents the frequency range of the sweep in octaves normalized to the signal length in seconds.

The length of the harmonic impulse responses has to be considered as well to avoid interference of the harmonics with the fundamental impulse response. For high sweep rates Δt_2 becomes so small that the second harmonic overlaps with the fundamental impulse response. This leads to a constraint for the sweep rate by using Eq. (1) and $k = 2$ as follows

$$r_{\mathrm{s}} \leq \frac{1}{\tau_{\mathrm{IR},2}}, \tag{2}$$

where $\tau_{\mathrm{IR},2}$ can be reasonably considered smaller than τ_{IR} for weakly nonlinear systems. Hence, a minimum length of the sweep τ_{sw} has to be considered. (Compare Eq. 6 in [12])

The measurement with an exponential sweep requires a certain time to allow the system to decay after the sweep has stopped. This time is introduced as a *stop margin* τ_{st}. Therefore, the measurement duration with this exponential sweep method with L loudspeakers measured in a sequential manner can be given as

$$T_{\mathrm{ES}}(L) = L \cdot (\tau_{\mathrm{sw}} + \tau_{\mathrm{st}}). \tag{3}$$

2 REVIEW OF THE MULTIPLE EXPONENTIAL SWEEP METHOD

2.1 Parallel Measurement

The multiple-exponential sweep method (MESM) proposed by MAJDAK ET AL. is applicable for weakly nonlinear systems [12]. This method reduces the measurement duration significantly compared to sequential measurements with the exponential sweep method when the number of sources or loudspeakers L is high. The sweeps are played back with a certain waiting time or delay τ_w between the sweeps. Hence, sweeps of several loudspeakers might run in (semi-) parallel manner as they partly overlap. In the ideal case without any non-linearities the measurement duration of L parallel exponential sweep (PES) measurements with a sweep of length τ_{sw} and a stop margin τ_{st} is given by[1]

$$T_{PES}(L) = (L-1)\tau_w + \tau_{sw} + \tau_{st}. \quad (4)$$

Please note that a different notation than in the original paper from MAJDAK ET AL. is used for improved readability of the proposed method.

Comparing the duration of the sequential measurement given by Eq. (3) with the duration of the overlapping method given by Eq. (4) results in

$$\frac{T_{PES}(L)}{T_{ES}(L)} = \frac{(L-1)\tau_w + \tau_{sw} + \tau_{st}}{L(\tau_{sw} + \tau_{st})}. \quad (5)$$

The theoretically achievable reduction of measurement time for an infinitely large number of loudspeakers can be expressed as

$$\lim_{L\to\infty} \frac{T_{PES}(L)}{T_{ES}(L)} = \frac{\tau_w}{\tau_{sw} + \tau_{st}}. \quad (6)$$

Usually the length of the excitation signals lies in the range of 0.2 s for very short and 2 s for moderately long sweeps. The parallel measurement speeds up the measurement especially for a large number of sound sources L, long sweeps and short delays between the sweeps. Hence, the minimization of this delay is of interest. A minimum time has to be waited for corresponding to the decay of the system to be measured according to

$$\tau_w \geq \tau_{IR}. \quad (7)$$

This parallel measurement method can yield impulse responses that have the same quality in terms of signal to noise ratio as separately measured impulse responses for LTI systems.

2.2 Overlapping

MAJDAK ET AL. introduced two different strategies to enhance this method's applicability to weakly non-linear systems [12]. One method is called *overlapping (OL)*, where the harmonic impulse responses appear between the impulse responses of interest as shown in Fig. 2. As a drawback of the occurrence of harmonic impulse responses the delay $\tau_{w,OL}$ has to be increased compared to Eq. (7) to not interfere with the impulse response of interest. Furthermore, the sweep rate can be increased to shorten the delay between fundamental and harmonics (also reducing the obtained SNR). The maximum order of harmonics k_{max} present in the measurement has to be finite and preferably small to allow a small $\tau_{w,OL}$. The illustrative examples in this paper use $k_{max} = 4$.

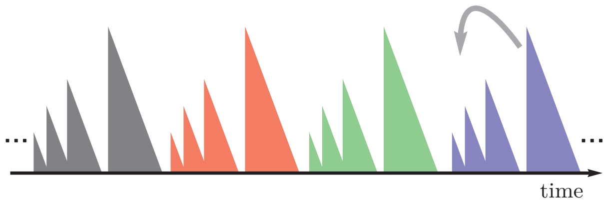

Fig. 2. Excitation strategy *overlapping* with high sweep rate.

The time delay τ_w has to fulfill

$$\tau_{w,OL} \geq \Delta t_{k_{max}} + \tau_{IR} = \frac{\log_2(k_{max})}{r_s} + \tau_{IR} \quad (8)$$

to avoid interference, where r_s still has to fulfill Eq. (2). This overlapping method can directly be used consecutively for an arbitrary number of loudspeakers L. Hence, the measurement duration can directly be given according to Eq. (4) but with $\tau_w = \tau_{w,OL}$ as described in Eq. (8).

2.3 Interleaving

The other strategy previously proposed is called *interleaving* (IL) [12], where η impulse responses of interest are grouped together to fit as many fundamental impulse responses in the time span between the first fundamental impulse response and its corresponding harmonic $k = 2$. This is illustrated in Fig. 3 for $k_{max} = 4$ and $\eta = 4$. Therefore the constraint from Eq. (2) has to be narrowed to

$$\Delta t_2 \geq \tau_{IR,2} + (\eta - 1)\tau_{w,IL} \quad (9)$$

with $\tau_{w,IL}$ the new time delay between the sweeps for this interleaving method that just has to fulfill Eq. (8).

The measurement duration with this method for a limited number of systems η is given as $T_{IL}(\eta) = T_{PES}(\eta)$ according to Eq. (4) by using $\tau_{w,IL}$.

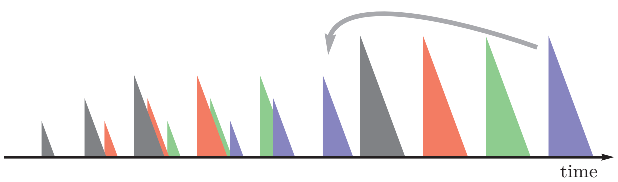

Fig. 3. Excitation strategy *interleaving* with low sweep rate and $\eta = 4$.

2.4 MESM

A combination of both strategies with two different time delays $\tau_{w,IL}$ and $\tau_{w,OL}$ is called the *multiple exponential sweep method*. It uses the overlapping method of inner multiple sweeps generated by the interleaving method. An illustration with $\eta = 2$ is shown in Fig. 4. An optimum

[1]This is the same formulation as later used for the *overlapping* technique but should be seen as a general formulation at this point. Instead of the exact time delay, when L is very large, a mean delay can be used instead to approximate the measurement duration of methods with mixed delays.

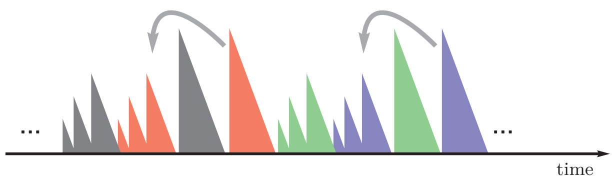

Fig. 4. Impulse response measured with the MESM ($\eta = 2$).

solution can be found by either using an optimization algorithm or a closed formulation. This method can also be used consecutively without additional time delays as was the case for the overlapping method.

WEINZIERL ET AL. proposed a generalization of the multiple sweep method by allowing deviating spectral shapes and individually designed instantaneous frequency over time functions [14]. Magnitude shaping of the sweeps to compensate for loudspeaker frequency responses can be applied to the MESM without loss of generality. The modification of the sweeps' group delay results in a blur of the harmonic impulse responses in time. A trade-off between the spectral shape of the SNR and an increased length of the harmonics is used to find an optimum solution. However, the authors state that, even though the original MESM mostly outperforms their proposed method for short decay times $t_{IR} < 0.1\,\mathrm{s}$, their method can become beneficial for $t_{IR} > 2\,\mathrm{s}$

The signal to noise ratio and the temporal and spectral structure of the results obtained by MESM and sequential measurements remain the same if the following requirements are fulfilled:

1. The system has to be weakly nonlinear only, i.e. the number of harmonic impulse responses has to be small;
2. In case non-linearities are observed, the level has to be kept constant between the actual measurement and a possible calibration measurement. Note that this constraint has not been stated in the original paper by MAJDAK ET AL.;
3. The length of the impulse response should be smaller than the smallest delay τ_w between two subsequent sweeps;
4. Once the weakly non-linear loudspeakers play back the MESM signal no further weak non-linearities are allowed, i.e., the microphones and preamplifiers have to be driven in a linear range only.

To obtain the minimum measurement duration

$$\tau_{w,IL} = \tau_{IR} \tag{10}$$

is chosen according to Eq. (8). The maximum value η for weakly non-linear systems ($\tau_{IR} \geq \tau_{IR,2}$) for a given sweep rate can be calculated as[2]

$$\eta = \left\lfloor \frac{\tau_{IR} - \tau_{IR,2} + {}^{1}\!/_{r_s}}{\tau_{IR}} \right\rfloor . \tag{11}$$

[2]Please note that this equation deviates from Eq. 15 in [12] as already published in [14].

The parameters required for the optimization of the MESM only depend on the length of the impulse response of the fundamental and first appearing harmonic ($k = 2$) and the maximum number of harmonics observed k_{max}. If L tends to infinity, a mean delay $\overline{\tau}_{w,MESM}$ between the sweeps can be given as

$$\overline{\tau}_{w,MESM} = \frac{(\eta - 1) \cdot \tau_{w,IL} + \tau_{w,OL}}{\eta}. \tag{12}$$

The measurement duration could then be approximated by Eq. (4) for high values of L with this delay. However, for the examples presented in the following section with limited L, the measurements durations for the MESM are calculated according to Eq. 13 in the original MESM paper by MAJDAK ET AL..

The theoretical minimum delay $\overline{\tau}_{w,MESM}$ can then be found for the maximum sweep rate $r_{s,max}$ for each η fulfilling the constraint in Eq. (11)

$$r_{s,max}(\eta) = \frac{1}{\tau_{IR}(\eta - 1) + \tau_{IR,2}}. \tag{13}$$

By inserting Eq. (13) into Eq. (12) the locally minimum delays for η can be expressed as

$$\overline{\tau}_{w,MESM,\eta} = \tau_{IR} + \log_2(k_{max}) \cdot \frac{\tau_{IR}(\eta - 1) + \tau_{IR,2}}{\eta}. \tag{14}$$

The minimum delay is always found for weakly nonlinear systems with $\tau_{IR,2} < \tau_{IR}$ for $\eta = 1$ as

$$\overline{\tau}_{w,MESM,min} = \tau_{IR} + \log_2(k_{max}) \cdot \tau_{IR,2}. \tag{15}$$

3 OPTIMIZED MESM

The proposed optimization of the MESM

- takes advantage of the temporal structure of the impulse response;
- uses a different placement strategy for the harmonic impulse responses than interleaving or overlapping by placing the single harmonics between arbitrary fundamentals;
- considers the amplitude of all significant harmonic impulse responses along with their SNR and with that a reduced length for each harmonics order.

This method uses just one time constant τ_w between the sweeps. Hence, the measurement duration can directly be expressed according to Eq. (4). The sweep parameters are chosen differently as explained next.

3.1 Temporal Structure of Impulse Response

The impulse response (IR) of an acoustic system can be reasonably assumed to be causal and have an energetically exponential decay of arbitrary kind. As for measurements of directional transfer functions of the *devices under test* (DUTs) with loudspeakers as sound sources, the obtained impulse responses consist of a direct sound path followed by reflections due to objects or e.g. room boundaries. The overall length of these impulse responses is described by

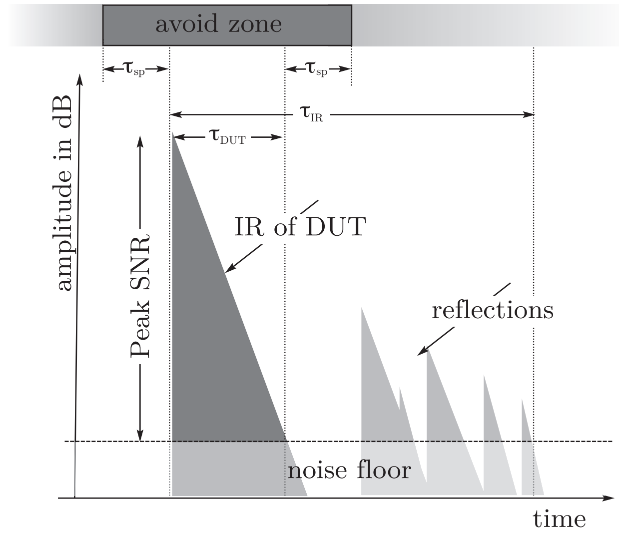

Fig. 5. Temporal structure of an impulse response measured with an exponential sweep.

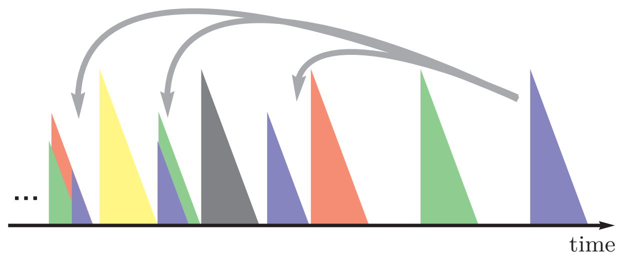

Fig. 6. Placement strategy of the optimized MESM.

τ_{IR} as introduced above. Even in anechoic environments such reflections might still occur caused by either a hard floor or necessary objects in the room, e.g. lamps, carriers, doors, pedestals. Hence, the impulse response can be described as shown in Fig. 5.

The direct sound contains the only important spectral and directional information if the reflections are not caused by the object itself and if the reflections arrive after the direct sound has decayed sufficiently. This important part of the impulse response with the length τ_{DUT} has to be protected against reflections and also against harmonic impulse responses that might overlay during a measurement with multiple simultaneous sweeps. Hence, an *avoid zone* around this impulse response of the DUT is introduced with an optional safety time τ_{sp} to the left and to the right leading to the ratio:

$$\alpha = \frac{\tau_{DUT} + 2\tau_{sp}}{\tau_{IR}} \leq 1 + \frac{2\tau_{sp}}{\tau_{IR}}. \qquad (16)$$

This ratio describes the percentage of the length of the impulse response that can be considered as useful. If the desired information is spread over the entire length of the impulse response and no additional safety time is required this approach is not beneficial ($\alpha = 1$). Values smaller than 1 are an indicator that this approach helps to decrease the measurement duration.

In the MESM the harmonic impulse responses were placed between the fundamental impulse responses of interest. Since the reflection do not carry valuable information these areas in the impulse response can be used to place the harmonic impulse responses. Hence, harmonic impulse responses can also be placed anywhere inside the time τ_{IR} except for the introduced avoid zone.

The overlapping method in general and hence also the MESM can directly benefit from this *adapted overlapping method (AOL)* by adapting Eq. (8) to

$$\tau_{w,AOL} = \frac{\log_2(k_{max})}{r_s} + \tau_{DUT} + \tau_{sp}. \qquad (17)$$

The MESM using this approach is called *adapted MESM* in the following. Eq. (10) and Eq. (11) remain unchanged. Thus, the measurement duration of the adapted MESM is always smaller than the original MESM for $\alpha < 1$.

3.2 Placement Strategies for Harmonic Impulse Responses

The harmonic impulse responses caused by a particular system do not need to gather in a group as in the case for interleaving and overlapping method. The only claim for the harmonic impulse responses is to not fall into the avoid zones as illustrated in Fig. 6.

From the linear case it is already known that the time delay between the sweeps has to fulfill $\tau_w \geq \tau_{IR}$ but it can directly by formulated a second constraint allowing a minimum space between two avoid zones. The combination of both constraints is given by

$$\tau_w \geq \max\left(\tau_{DUT} + 2 \cdot \tau_{sp} + \max(\tau_{IR,k}), \tau_{IR}\right) \qquad (18)$$

narrowing the search range. The start of each harmonic k has to fall after the end of an avoid zone,

$$(-\Delta t_k \bmod \tau_w) \geq \tau_{DUT} + \tau_{sp}. \qquad (19)$$

and for the end to appear before the next avoid zone starts,

$$(-\Delta t_k \bmod \tau_w) + \tau_{IR,k} \leq \tau_w - \tau_{sp} \qquad (20)$$

leading to a combined constraint by using Eq. (1)

$$\tau_{DUT} + \tau_{sp} \leq -\frac{\log_2(k)}{r_s} \bmod \tau_w \leq \tau_w - \tau_{sp} - \tau_{IR,k}. \qquad (21)$$

Two parameters are to be found to fulfill the constraint in Eq. (21) for the avoid zone: τ_w and r_s. It should be pointed out that a change in r_s will alter the obtained overall SNR.

3.3 Amplitude and Length of Harmonics

Systems considered in this work are weakly nonlinear systems that can be reasonably quantified by claiming a value for total harmonic distortion below 10 % for all frequencies. This results in an attenuation of all harmonics of at least 20 dB. Harmonic impulse responses obtained with the exponential sweep method show the same exponential decay rates as the fundamental impulse response. This leads to lengths $\tau_{IR,k}$ of the harmonics always shorter than the length of the fundamental impulse response to be considered for the calculation of the optimum parameters. This can be explained by the fact that, when the decay rate of the harmonics remains unchanged, the harmonic impulse responses with lower energy disappear more and more into the noise floor. The maximum order k_{max} and the length of each harmonic impulse response can be obtained from a calibration measurement using subsequent single sweep measurements.

The values for $\tau_{IR,k}$ depend on the signal to noise ratio. It has to be pointed out that once an SNR is chosen for the calculation of these values this SNR cannot be exceeded in the ongoing measurements even with higher amplitudes or averaging. The influences of these parts below the noise floor of the harmonic impulse responses on the fundamental impulse response are still correlated in contrast to the noise itself. Harmonics that do not show a value $\tau_{IR,k} > 0$ can be neglected.

There exist two methods in obtaining $\tau_{IR,k}$. The first method is straightforward and measures these times along with the amplitude of the harmonic impulse responses in time domain with the excitation level used for the ongoing measurements. As a drawback, it cannot be guaranteed that the harmonics are kept below a certain limit for all frequencies as the time domain just gives an average attenuation.

The second method just uses the length τ_{IR} of the fundamental and the minimum spectral attenuation of the harmonics. This attenuation can be found in frequency domain as the minimum difference a_k in dB between the spectrum of an harmonic k compared to the spectrum of the fundamental. [3, 5].

By assuming the same decay rate of the harmonics and the fundamental the formulation holds

$$\tau_{IR,k} = \frac{\text{SNR} - a_k}{\text{SNR}} \tau_{IR} \qquad (22)$$

with the peak SNR in dB in time domain as a worst case approximation. This approach uses a nonlinear acoustic model based on [17] and separates the influence of the nonlinearities caused by the loudspeaker (a_k) and the decay of the room (τ_{IR}).

For the MESM only $\tau_{IR,2}$ was required. With increasing order of harmonics the amplitude of the harmonics are expected to decrease leading to even shorter $\tau_{IR,k}$. The proposed placement strategy can benefit from this behavior. The influence is studied in simple manner only by using just a single value for a_k and hence The influence is studied in a simple manner only by using just a single value for a_k and hence the same lengths for the harmonics $\tau_{IR,k}$ with $k \geq 2$. However, the lengths of the harmonics might still be determined by analyzing a calibration measurement in time domain.

3.4 Optimization of the Parameters

To the best knowledge of the authors there is no analytic solution to find the optimum values for τ_w and r_s to fulfill Eq. (18) and Eq. (21) for arbitrary values of α, k_{max} and $\tau_{IR,k}$. Therefore an exhaustive search method is applied. The minimum τ_w found also gives the minimum measurement duration for the case when $L \to \infty$.

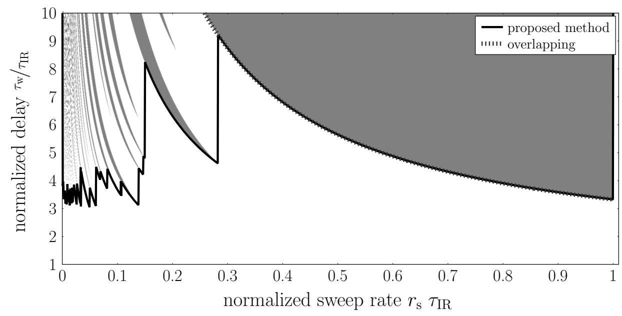

Fig. 7. Valid combinations to fulfill the constraints for the adapted overlapping method for $k_{max} = 4$, $\alpha = 1$, $\tau_{IR,k} = \tau_{IR}$ (white: interference of harmonics with fundamentals, gray: no interference, black: minimum delay between sweeps).

It has to be pointed out that the combination of a minimum τ_w and a very low r_s does not lead to the interleaving method with $\eta = L$ since the constraint in Eq. (21) is fulfilled for an infinite number of sound sources and only a single time delay is used in the proposed method.

The two dimensional search space (r_s,τ_w) can be transformed to a normalized search space ($r_s \cdot \tau_{IR}$,τ_w / τ_{IR}) that becomes independent on the length of the impulse response τ_{IR}. General constraints for the search space are given by Eq. (2) and Eq. (7).

Valid combinations are shown in Fig. 7 for an example with $k_{max} = 5$, $\alpha = 1$ and no decrease of the harmonics: $\tau_{IR,k} = \tau_{IR}$. The minimum time delay possible between the sweeps is of interest. Valid combinations can always be found for high sweep rates and long delays since the method is equivalent to the overlapping method in this area according to Eq. (8). It can be stated that this placement scheme results in time delays equal to or shorter than the overlapping method. Towards lower sweep rates the range of valid delays resulting in valid solutions become smaller. As small deviations in the actual sweep rate of the excitation signal from this optimum value directly results into a violation of the avoid zone constraint, the use of these small sweep rates should be avoided. Furthermore, the delay will be likely choosen as an integer number of samples and this deviation might as well lead to a violation of the avoid zone constraint.

Due to a strong fluctuation of the minimum delay over the sweep rate observed in Fig. 7, it becomes evident that the sweep rate should not be fixed prior to the search. A range for the sweep rate might be set in advance and the sweep rate with the lowest delay possible can be chosen instead. This leads directly to a small change of the SNR that can be neglected in most cases.

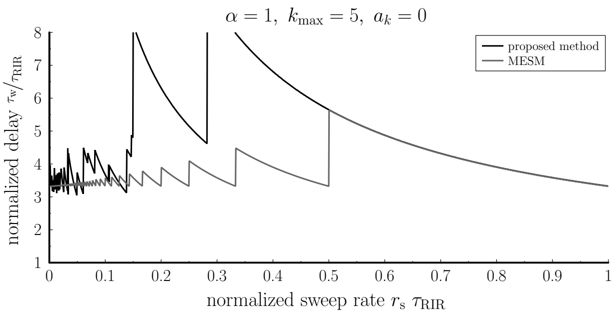

Fig. 8. Comparison of minimum normalized delay in MESM and proposed method (only placement strategy, $\tau_{IR,k} = \tau_{IR}$).

4 Comparison with MESM

The influence of the three different optimizations introduced above is first discussed for the different placement strategy only and then separately for the remaining optimizations in combination with the placement strategy. Please note that the order of appearance of the optimization contributions deviates from that in Section 3.

4.1 Placement Strategy

The MESM does not originally consider the temporal structure of the impulse response in a way that it distinguishes between relavant and irrelevant parts of the impulse response. Hence, the measurement duration with the proposed method without this consideration is compared to the original MESM by setting $\tau_{IR} = \tau_{DUT}$. The maximum number of harmonics is chosen to be $k_{max} = 5$ and $\tau_{IR,k} = \tau_{IR}$ which is a worst case scenario for typical loudspeakers used for this kind of measurements. With decreasing sweep rate the curve of the MESM shows jumps. These directly correspond to an increase of η. For high normalized sweep rates (above $1/2$) the interleaving method uses $\eta = 1$ and the MESM is therefore equivalent to the overlapping method. As can be seen in Fig. 8, both methods always result in a minimum normalized delay shorter or equal to the delay obtained with just the overlapping method presented in Fig. 7. The proposed method shows slightly lower values for some low normalized sweep rates only. It can be concluded that the adapted overlapping method and therefore the proposed placement strategy alone does not directly allow much faster measurement durations and is only beneficial in a very narrow range of sweep rates.

4.2 Temporal Structure with Placement Strategy

The influence of the parameter α discribing the ratio of the relevant part of the impulse response to its entire length is studied in Fig. 9. The original and the adapted formulation using Eq. (17) for the MESM is used to plot Eq. (12). As can be seen, with decreasing percentage of the length of the important part of the impulse response the proposed method outperforms even the adapted MESM for nearly all normalized low sweep rates. It can be stated that the placement strategy is only advantagegous for low values of α and low sweep rates where the placement of the harmonics reaches over the time span of various impulse responses.

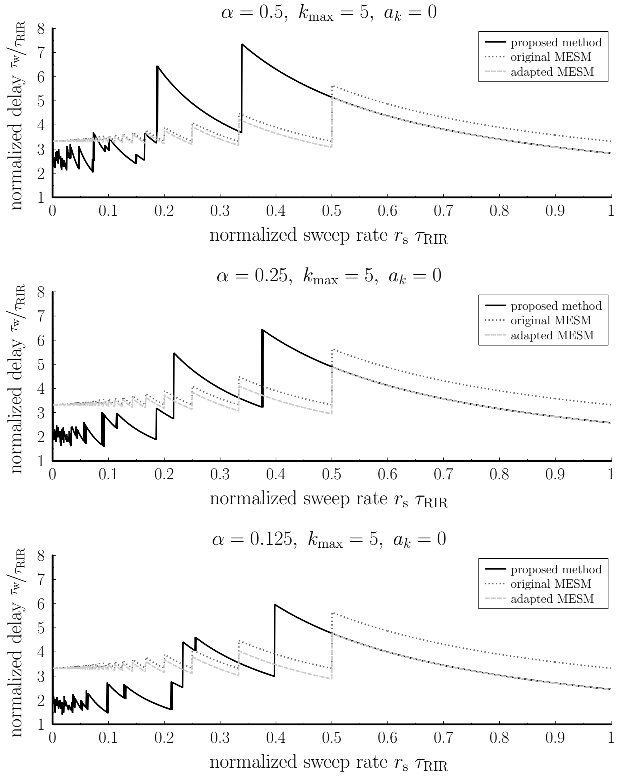

Fig. 9. Comparison of minimum normalized delay in MESM and proposed method for different α (ratio of relevant part of DUT to entire impulse response length).

4.3 Lengths of Harmonics with Proposed Placement Strategy

The influence of the lengths of the harmonics is studied along with the placement strategy in Fig. 10. This reduction in length is calculated using the attenuation a_k which is chosen to be 20 dB and 40 dB for every k. As a reference curve the delays for the MESM without adaptation and without using the reduced values labeled *reference MESM* for $\tau_{IR,2}$ are introduced. The values for the original MESM using this reduction only differ in the vicinity of the jumps. This can be explained by the fact that η can already be choosen a step higher for these sweep rates due to a reduced $\tau_{IR,2}$. Hence, the possible minimum delays for the adapted MESM are smaller in these regions than for the original MESM. The proposed method yields shorter delays for low sweep rates only and becomes advantageous with shorter lengths of the harmonics.

4.4 Combination with Realistic Values

The interaction of the proposed placement strategy with both temporal structure and the lengths of the harmonics is studied in Fig. 11. Realistic values have been choosen for α and a_k. These values are also used in one of the following application examples. The minimum delays for the MESM without taking advantage of the decreased length of the

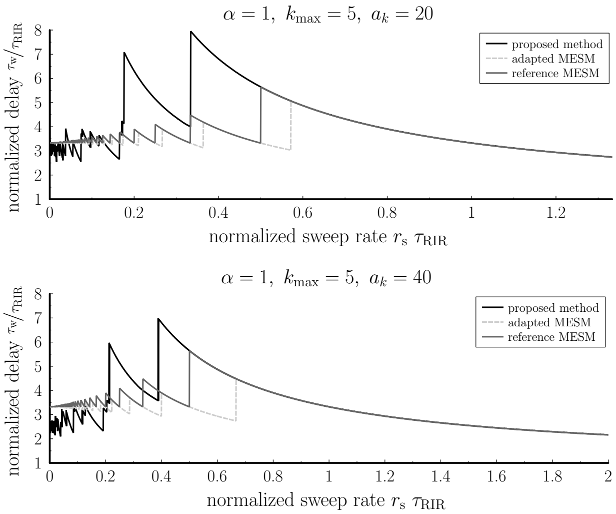

Fig. 10. Comparision of the minimum delay for different a_k for the proposed method with the original and a reference MESM which does not use a reduced length for the harmonics.

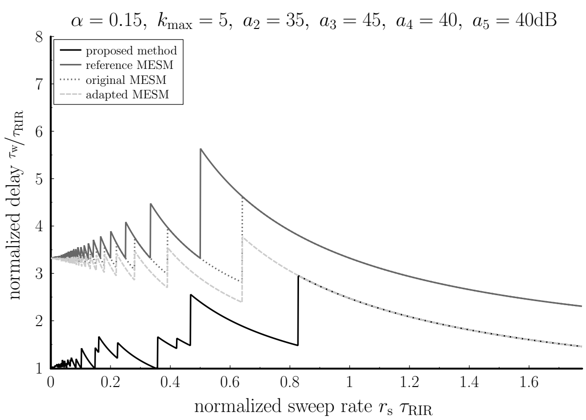

Fig. 11. Comparision of delay for proposed method with realistic values for α and a_k and the reference and adapted MESM.

harmonics and without the adaptation introduced before are again label as reference MESM. The original MESM consideres the reduced length for the harmonics only. The adapted MESM consideres both effects. Obviously, the adapted MESM always outperforms the reference MESM. The proposed method outperforms the adapted MESM for all normalized sweep rates below approx. 0.83. Some normalized delays are very close or even equal to the theoretical minimum of 1.

5 APPLICATION

Two different examples for application of the parallel measurement method along with the speed-up achieved are presented in this section. A summary of the calculated values is given in Table 5. Details on the implementation and the source code are given in the appendix A.4

Table 1. Application Examples of Parallel Measurement Methods

	Directivity	WFS
L	40	192
τ_{IR}	40 ms	1 s
τ_{DUT}	4 ms	50 ms
τ_{sp}	1 ms	1 ms
τ_{st}	40 ms	1 s
α	0.15	0.05
Separate Measurement	55.209 s	1001.1 s
(sweep duration)	1.340 s	4.2 s
MESM	5.934 s	445.7 s
Adapted MESM	5.682 s	264.2 s
(sweep duration)	1.367 s	4.2 s
Proposed method	3.256 s	226.6 s
(sweep duration)	1.340 s	23.4 s

5.1 Measurement of Directivity

As a first application the measurement of a directional microphone with a prominent radiation pattern—at least at higher frequencies—is chosen, similar to the example in [12]. This microphone is surrounded by $L = 40$ broadband loudspeakers mounted into a measurement arc as described in [15]. The impulse response of the loudspeaker and the microphone is short with an approximate duration of $\tau_{DUT} = 4\,\text{ms}$ [16]. On the other hand, the impulse response of the hemi-anechoic chamber of the institute used for the measurements shows reflections from the floor, supports, mounts and doors in the order of $\tau_{IR} = 40\,\text{ms}$. The avoid zone was enlarged by $\tau_{sp} = 1\,\text{ms}$. Hence, it follows $\alpha = 0.15$. The stop margin was safely choosen as $\tau_{st} = \tau_{IR}$. Sufficient SNR could be achieved in a test measurement with $\tau_{sw} = 1.5\,\text{s}$ yielding more than 80 dB peak SNR in the impulse response. The frequency range of interest goes from 100 Hz to 18 kHz. This corresponds to a sweep rate $r_s = 5$ or a normalized sweep rate $r_s\tau_{IR} = 0.2$ (See Fig. 11 for details on the parameters). The maximum harmonic order was $k_{\max} = 5$ and a_k based on a distortion measurement with exponential sweeps with the same level as in the final measurements: $a_2 = 35\,\text{dB}$, $a_3 = 45\,\text{dB}$, $a_4 = 40\,\text{dB}$ and $a_5 = 40\,\text{dB}$.

According to Eq. (11) $\eta = 5$ is obtained for $r_s = 5$. The highest sweep rate to fulfill Eq. (11) for $\eta = 5$ can be found as $r_{s,\max} = 5.48$ and the sweep duration follows as 1.3673 s. For this sweep rate the mean delay $\overline{\tau}_s$ has a local minimum and along with the higher sweep rate this yields the fastest measurement in this region. The change in SNR is calculated as $-1.5\,\text{dB}$. As interleaving delay constant the minimum τ_{IR} is used. The remaining parameter for the MESM as used in the original paper—the additional time required for each interleaving set, see [12] for more details—has been calculated as $\tau'_k = 0.4238\,\text{s}$. The measurement with the original MESM takes 5.934 s. The optimum values for the sweep rate can also be observed in Fig. 11 for the normalized sweep rate 0.219 for the MESM curve.

The adapted MESM uses the same MESM parameters except for $\tau'_k = 0.3878\,\text{s}$. This corresponds to curve label adapted MESM in Fig. 11. The measurement duration can be reduced to 5.682 s.

The best combination of sweep rate and delay for the proposed method was found with the optimization algorithm in the vicinity of $r_s = 5$ as $r_{sw,opt} = 5.59$ ($r_{sw,opt} \cdot \tau_{IR} = 0.2236$) and $\tau_{w,opt} = 48.095\,\text{ms}$. Without considering shorter lengths for the harmonics ($a_k = 0\,\text{dB}$) the optimum solution could be found as $r_{sw,opt} = 5.26$ and $\tau_{w,opt} = 65.057\,\text{ms}$. This solution results in a longer duration as the delay is longer and the sweep length is longer due to a lower sweep rate. Hence, the first result is used. The new sweep has a length of 1.34 s and the theoretical change in SNR is calculated as $\Delta\text{SNR} = 10\log_{10}\left({}^{r_s}/_{r_{sw,opt}}\right) = -0.48\,\text{dB}$. For the given measurement setup with $L = 40$ loudspeakers the measurement duration was reduced from 55.209 s for the separate measurement to 3.256 s for the measurement with the proposed method.

The comparison of the results in frequency domain obtained with the proposed method and by separate measurement reveals maximum deviations of $\pm 0.1\,\text{dB}$ over the entire frequency range of interest. Deviations in the same order of magnitude are also observed when comparing results obtained from distinct measurements using the same method. Hence, the proposed method—in the same manner as the MESM—does not introduce noticeable errors if the requirements introduced above are met.

The measurement duration can therefore be reduced to 5.9 % with the proposed method compared to separate measurements. The theoretical limit for the reduction for the proposed method is 3.5 % according to Eq. (6). This could only be achieved by increasing the number of loudspeakers for parallel excitation. The theoretical limit for the adapted MESM for the given configuration is 8.5 %.

5.2 Condition Monitoring of Loudspeaker Array

As a second example, condition monitoring of an installed large loudspeaker array, e.g. for wave field synthesis in a cinema, with $L = 192$ loudspeakers is chosen. The purpose of such a measurement is to verify the correct functioning of the different drivers in each loudspeaker. This could be achieved by e.g. comparing the obtained frequency response with a reference curve. Hence, the direct sound component of the room impulse response carries all relevant information. Along with an appropriate microphone array the position of the loudspeakers might be monitored as well by using the time of arrival in the direct sound. The room impulse response decays into the noise floor after $\tau_{IR} = 1\,\text{s}$ and the stop margin is chosen safely as $\tau_{st} = 1\,\text{s}$. The maximum distance from the loudspeakers to the microphone is 12 m. The maximum length of the loudspeakers' impulse response is assumed as less than 10 ms. The part of the direct sound in the impulse response carrying the information of the DUT is generously approximated by $\tau_{DUT} = 50\,\text{ms}$ and thus $\alpha = 0.05$ ($\tau_{sp} = 0$).

The same assumptions for non-linearities and the frequency range are made as in the previous example. This results in $\tau_{IR,2} = 0.56$ and therefore $r_{s,max} = 1.778$ according to Eq. (2). For the separate measurements the shortest overall measurement duration is achieved by using the highest sweep rate possible as the SNR is not that critical. The length of the sweep is calculated as 4.2 s and the measurement for all loudspeakers takes 1001.1 s.

The optimum MESM parameters are found with a sweep rate $r_s = 1.78$, $\eta = 1$, a sweep length of 4.2 s, $\tau'_k = 1.3061\,\text{s}$. Hence, the measurement duration is 445.7 s. The measurement duration for the adapted MESM is 264.2 s with the same MESM parameters except for $\tau'_k = 0.356\,\text{s}$.

The optimum solution for the proposed method is found with $r_s = 0.32$ and $\tau_w = 1.0583\,\text{s}$. Hence, the theoretical minimum delay of the linear case is almost achieved with the proposed method. The overall measurement duration reduces to 226.6 s therefore to 22.6 % of the time required for the sequential measurement procedure. This value is already very close to the theoretical limit of 20.3 % according to Eq. (6) with the short sweep length. The theoretical limit for the adapted MESM is 26.0 %. It has to be pointed out that the focus does not lie on the SNR in this example but only on the fast measurement without artifacts by harmonic impulse responses. Hence, different optimum sweep lengths are used for the different methods. However, the proposed method delivers the fastest measurements although it uses the longer sweeps and hence yields results with higher SNR than the other methods.

6 CONCLUSION AND PERSPECTIVES

An optimized placement strategy for the harmonic impulse responses for the multiple exponential sweep method has been proposed. The placement strategy of the original method using two different delay constants between the sweeps can therefore be avoided. The proposed method can yield shorter delay constants improving the speed of the measurement. The proposed method can outperform the previous method by further consideration of the structure of the measured impulse response. Under certain conditions, especially when only a small percentage of the measured impulse response is of interest, the proposed method yields delays only slightly higher than that obtained in the ideal linear case. It has been shown that the formerly introduced overlapping method is a special case of the proposed method, whereas the proposed overlapping method yields the same or even shorter delays. Theoretical limitations for the delay and sweep rate have been introduced in a graphical manner using a normalized two dimensional search space. The used of the actual (smaller) length of the harmonic impulse responses instead of the lenght of the fundamental impulse response in the parameter optimization procedure has been proven beneficial.

The proposed method could also be used in combination with the original interleaving method. The mean delay can be even smaller than the original MESM, but with the drawback of again using two different delays making the optimization process and measurement post-processing more complex. However, the possible speed-up is assumed to be negligible as the delay for the proposed method is sometimes already close to the theoretical minimum.

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APPENDIX

A.1 Formulation of Sweep

The following formulation is based on [3, 17, 19]. A sweep signal is generally defined in time domain by

$$s(t) = \sin(\phi_{\mathrm{sw}}(t) + \phi_0) \tag{A.1}$$

with its time varying phase component $\phi_{\mathrm{sw}}(t)$ and the starting phase ϕ_0. Synonyms used in literature are: chirp or swept-sine. It is convenient to choose the starting phase $\phi_0 = 0$ as this results in a smooth start of the signal without a jump. The instantaneous frequency $f_{\mathrm{inst}}(t)$ over time is defined as [17]

$$f_{\mathrm{inst}}(t) = \frac{1}{2\pi}\frac{\mathrm{d}\phi_{\mathrm{sw}}(t)}{\mathrm{d}t}. \tag{A.2}$$

Commonly this instantaneous frequency is chosen as either a linear or an exponential function over time. An exponential—or sometimes called logarithmic—sweep starts at its lowest frequency f_1 and increases the frequency until its highest frequency f_2 in an exponential manner over time defined by the *sweep rate*:

$$r_{\mathrm{s}} = \frac{\log_2\left(f_2/f_1\right)}{\tau_{\mathrm{sw}}}\,\mathrm{octaves/s} \tag{A.3}$$

with the time τ_{sw} between those frequencies. As the following formulation uses the basis e instead of the basis 2 the rising time constant L_{s} is introduced as

$$L_{\mathrm{s}} = \frac{\log_2(\mathrm{e})}{r_{\mathrm{s}}} = \frac{\tau_{\mathrm{sw}}}{\ln\left(\frac{f_2}{f_1}\right)}. \tag{A.4}$$

The instantaneous frequency is therefore given as

$$f_{\mathrm{inst}}(t) = f_1 \cdot \mathrm{e}^{t/L_{\mathrm{s}}} \quad \text{and} \quad t \in [0, T] \tag{A.5}$$

and is zero otherwise. The phase of the sweep can now be obtained by integration of Eq. (A.2) as

$$\phi_{\mathrm{sw}}(t) = 2\pi \int_0^t f_{\mathrm{inst}}(\tau)\,\mathrm{d}\tau = 2\pi f_1 L_{\mathrm{s}} \left(\mathrm{e}^{t/L_{\mathrm{s}}} - 1\right). \quad \text{(A.6)}$$

A.2 Inverse Filter—Deconvolution

In order to obtain the impulse response $h(t)$ of an LTI system the system output $g(t)$ has to be deconvolved with the input signal $s(t)$. Despite the time domain methods, this deconvolution can be generally realized efficiently in frequency domain by transforming input and output signal with the Fourier transform ($S(f)$ and $G(f)$, respectively) and processing a spectral division to obtain the complex transfer function of the system

$$H(f) = \frac{G(f)}{S(f)} \quad \text{and} \quad S(f) \neq 0. \quad \text{(A.7)}$$

Problems arise for frequencies where the values of $S(f)$ become very small. As the output of the system is always superposed by measurement noise this noise will be amplified by the division leading to errors in the impulse response. This becomes evident as the exponential sweep only provides significant amplitude in the frequency range between f_1 and f_2.

In the digital domain the Discrete Fourier Transform is used instead. This directly leads to a cyclic convolution or deconvolution of multiplication or division is used in frequency domain. Generally, this periodicity might become problematic when the system shows non-linear behavior as harmonic impulse responses might overlap with the fundamental impulse response. Zero padding the discrete time signals s and g in the end and doubling the length of these signal prior to the devision can therefore solve this problem. This approximates *linear deconvolution*.

Especially for sweeps the *inverse sweep* can also be calculated directly in time domain as e.g. formulated in [17]. This inverse sweep is then convolved with $g(t)$ and no further regularization to account for the band limiation is required and it already has the same frequency limits as the original sweep.

A.2.1 Zero-phase Regularization

To overcome this problem FARINA [4] introduced a regularization method already used by KIRKEBY and many others in different contexts. The regularized inverse reads as follows

$$S_{\mathrm{inv,reg}}(f) = \frac{S^*(f)}{S^*(f)S(f) + \varepsilon(f)} \quad \text{(A.8)}$$

where $S^*(f)$ denotes the complex conjugate of $S(f)$ and $\varepsilon(f)$ is a real-valued and frequency dependent regularization parameter. The regularized transfer function follows accordingly as

$$H_{\mathrm{reg}}(f) = \frac{G(f)}{S(f)} \frac{1}{1 + \varepsilon(f)/|S(f)|^2} = \frac{G(f)}{S(f)} \cdot A_{\mathrm{reg}}(f), \quad \text{(A.9)}$$

where $A_{\mathrm{reg}}(f)$ describes the influence of the regularization as a filter. Since the input signal is band limited it is reasonable to increase the regularization parameter below f_1 and above f_2. The resulting transfer function is also band limited and is less influenced by measurement noise. The phase of $A_{\mathrm{reg}}(f)$ is zero for all frequencies but this filter is band limited. Hence, the equivalent impulse response of $A_{\mathrm{reg}}(f)$ is symmetric regarding the time axis, i.e. the impulse response has an non-causal part dependent on the band limiting frequencies f_1 and f_2.

Please note that the use of the inverse sweep along with a convolution also introduces a frequency band limitation. Depending on the particular implementation of time-discrete sweeps with finite lengths the frequency response of the convolution of the sweep with the inverse sweep might include ripples at the edges of the frequency range (Gibbs phenomenon). The phase of this frequency response is zero for all frequencies and hence this ideal impulse response is always symmetric due to band limitation and hence a-causal.

A.2.2 Minimum-phase Regularization

To overcome the problem of non-causality, $A_{\mathrm{reg}}(f)$ can be factorized into a minimum-phase (MP) regularization filter and a remaining non-causal all-pass (AP) filter [20, 21]:

$$A_{\mathrm{reg}}(f) = A_{\mathrm{reg,MP}} \cdot A_{\mathrm{reg,AP}}(f). \quad \text{(A.10)}$$

By using $A_{\mathrm{reg,MP}}$ in the deconvolution process the obtained fundamental impulse responses are always causal. These can be further processed by time windows. As a last step of data post-processing the all-pass component can be applied to compensate for the phase error in the pass-band yielding non-causal impulse responses again. This method has great advantages when the arrival time of an impulse is known and the time window should be applied directly at these pre-calculated arrival times. In the context of multiple excitation signals the measured and deconvolved signal have to be split into the impulse responses of separate systems. Non-causal impulse responses are problematic as the signal can spread to the left on the time axis interfering with the end of impulse response of the previous system. Hence, the split post-processing as described above is preferably used or at least a time shift accounting for this portion of the impulse response should be used instead.

A.3 Harmonic Impulse Responses

The output of a non-linear system to a monochromatic input signal, e.g. a pure tone or sine with a frequency f_0, is a superposition of pure tones with multiples of this fundamental frequency $f_n = n \cdot f_0$ and $n \in \mathbb{N}$ [3, 5]. For exponential sweeps the output of a non-linear system shows a superposition of harmonics of the sweep.

The dependency of the instantaneous frequency of the harmonics remains exponentially over time. But this slew rate is now a multiple of the original sweep. MÜLLER ET AL. [3] and FARINA [5] showed that this results in "time shifted" versions of the original sweep that appear as impulse responses of these harmonics after deconvolution. Therefore, the time shift Δt_k of the k-th harmonic can be found by solving:

$$f_{\text{inst}}(t + \Delta t_k) = k \cdot f_{\text{inst}}. \tag{A.11}$$

By inserting Eq. (A.2) it reads

$$f_1 \mathrm{e}^{\frac{t+\Delta t_k}{L_\mathrm{s}}} = k \cdot f_1 \mathrm{e}^{\frac{t}{L_\mathrm{s}}}. \tag{A.12}$$

Dividing by f_1, applying the natural logarithm and resolving to Δt the final results is

$$\Delta t_k = \ln(k) \cdot L_\mathrm{s} = \frac{\ln(k)}{r_\mathrm{s} \ln(2)} = \frac{\log_2(k)}{r_\mathrm{s}}. \tag{A.13}$$

NOVAK showed that the harmonic impulse responses are generally not in phase with the fundamental impulse response [22, 17]. This can be explained by the fact that only the instantaneous frequency was used to find the time shift. As the position of the harmonics strongly depends on the sweep rate it is necessary to generate sweeps with exact sweep rates.

A.4 Implementation

All routines used in this manuscript have been implemented in MATLAB using the ITA-Toolbox. The ITA-Toolbox is open-source and freely available with a BSD license under www.ita-toolbox.org. This toolbox provides special objects for audio signals. These objects allow various plots with automatic and normalized transformation between time and frequency domain. Functions for various signal processing tasks commonly used in acoustic are available that work directly on these audio objects.

Furthermore, it contains an application for the measurement of acoustic systems using standard audio sound boards under Windows, Linux and Mac OS. Current system and software requirements are listed on the web page. The measurement application features an object oriented approach that allows for automatic measurements of impulse responses including deconvolution and level compensation. The signal excitation and deconvolution techniques described in this appendix can all be configured accordingly with these measurement objects.

Routines for the optimization of the parameters for the proposed method are included with an additional script that was used to optimize the parameters for the given configurations used in this paper. A special measurement object for the MESM and the proposed method allowing semi-parallel excitation with arbitrary time delays between the channels of the driving loudspeaker is available. It provides an automatic measurement according to the specified parameters with subsequent partitioning of the impulse responses according to the loudspeaker channels and the time delays used.

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